CS 237: Probability in Computing

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Lecture 3: Problem Solving Methods with Decision Trees

- Review and finishing up from last time:
- Finite Probability, Equiprobable Case, Non-Equiprobable Case
- Decision Trees for Probability Spaces
- The Monte Hall Problem

Review: Finite Probability Spaces

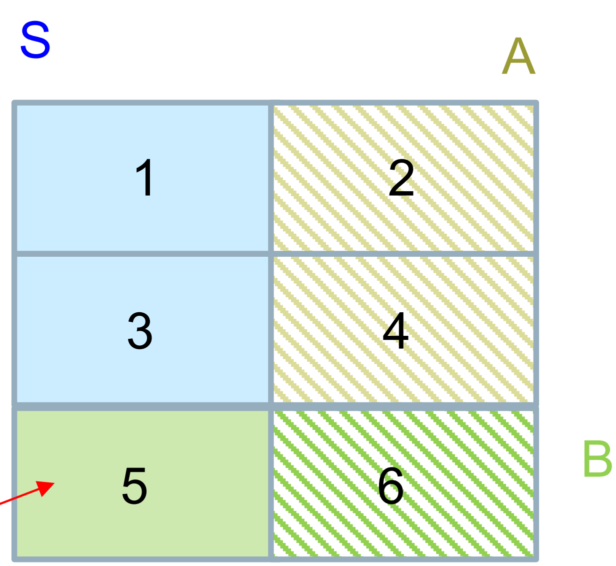
For finite probability spaces, it is easy to calculate the probability of an event; we just have to apply axiom P_3:

If event $\mathbf{A = \{ a_1, a_2, ..., a_n \}}$, then

$$\mathbf{P(A) = P(\{ a_1, a_2, ..., a_n \}) = P(a_1) + P(a_2) + ... + P(a_n)}$$

Example: Toss a die and output the number of dots showing. Let A = "there are an even number of dots showing" and B = "there are at least 5 dots showing."

We can illustrate simple problems by using the "area" = "probability" analogy:



Equiprobable: area of each elementary event is 1/6 = 0.16666...

$$P(A) = P(2) + P(4) + P(6) = 1/6 + 1/6 + 1/6 = ½$$

$$P(B) = P(5) + P(6) = 1/6 + 1/6 = 1/3$$

Review: Finite Probability Spaces

Example: Flip three fair coins and count the number of heads. Let A = "2 heads are showing" and B = "at most 2 heads are showing."

The equiprobable "pre-sample space" is

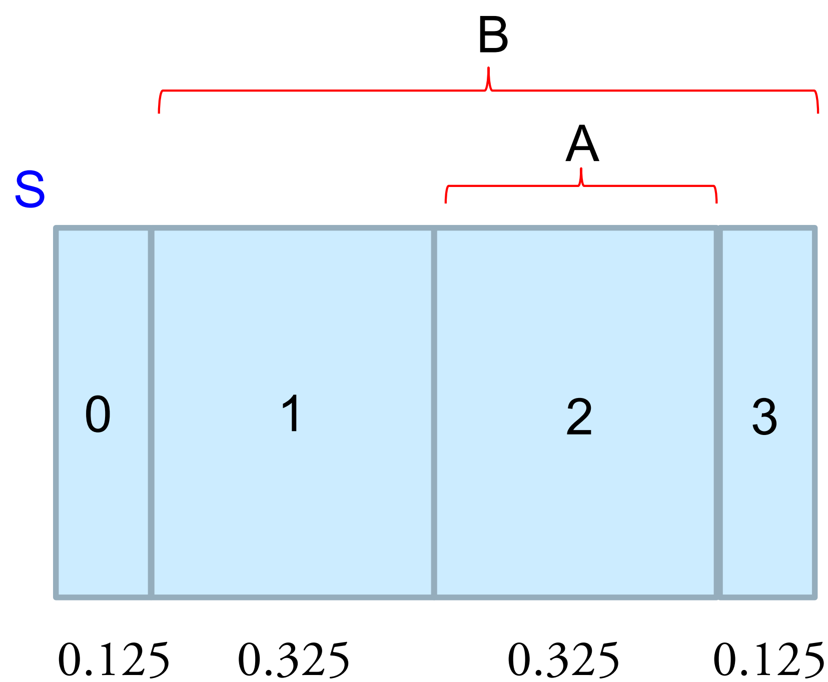
configuration:	**{ TTT,**	**TTH,**	**THT,**	**THH,**	**HTT,**	**HTH,**	**HHT,**	**HHH }**
# heads:	**0**	**1**	**1**	**2**	**1**	**2**	**2**	**3**

S = { 0, 1, 2, 3 }
P = { 1/8, 3/8, 3/8, 1/8 }

P(A) = P(2)
= 3/8

P(B) = P(0) + P(1) + P(2)
= 1/8 + 3/8 + 3/8
= 7/8

Not Equiprobable: area of each elementary event is different:



Finite Equiprobable Probability Spaces

For finite and equiprobable probability spaces,

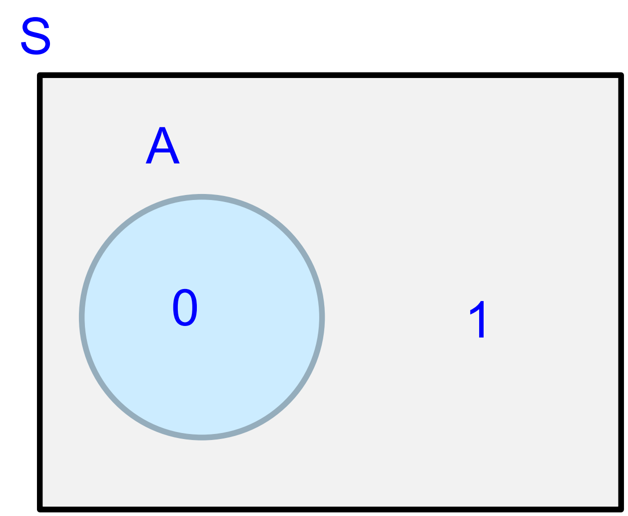
it is easy to calculate the probability:

|A| = cardinality of set A
= number of members

$$P(A) = \frac{|A|}{|S|}$$

Here, "area" = "number of elements."

Example: Flip a coin, report how many heads are showing? Let A = "the coin lands with tails showing"



S = { 0, 1 }

P = { ½, ½ }

Finite Equiprobable Probability Spaces

For finite and equiprobable probability spaces,

it is easy to calculate the probability:

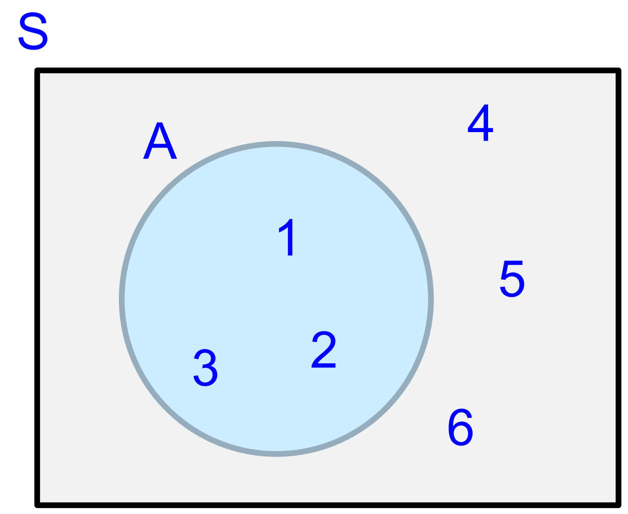
$$P(A) = \frac{|A|}{|S|}$$

Here, "area" = "number of elements."

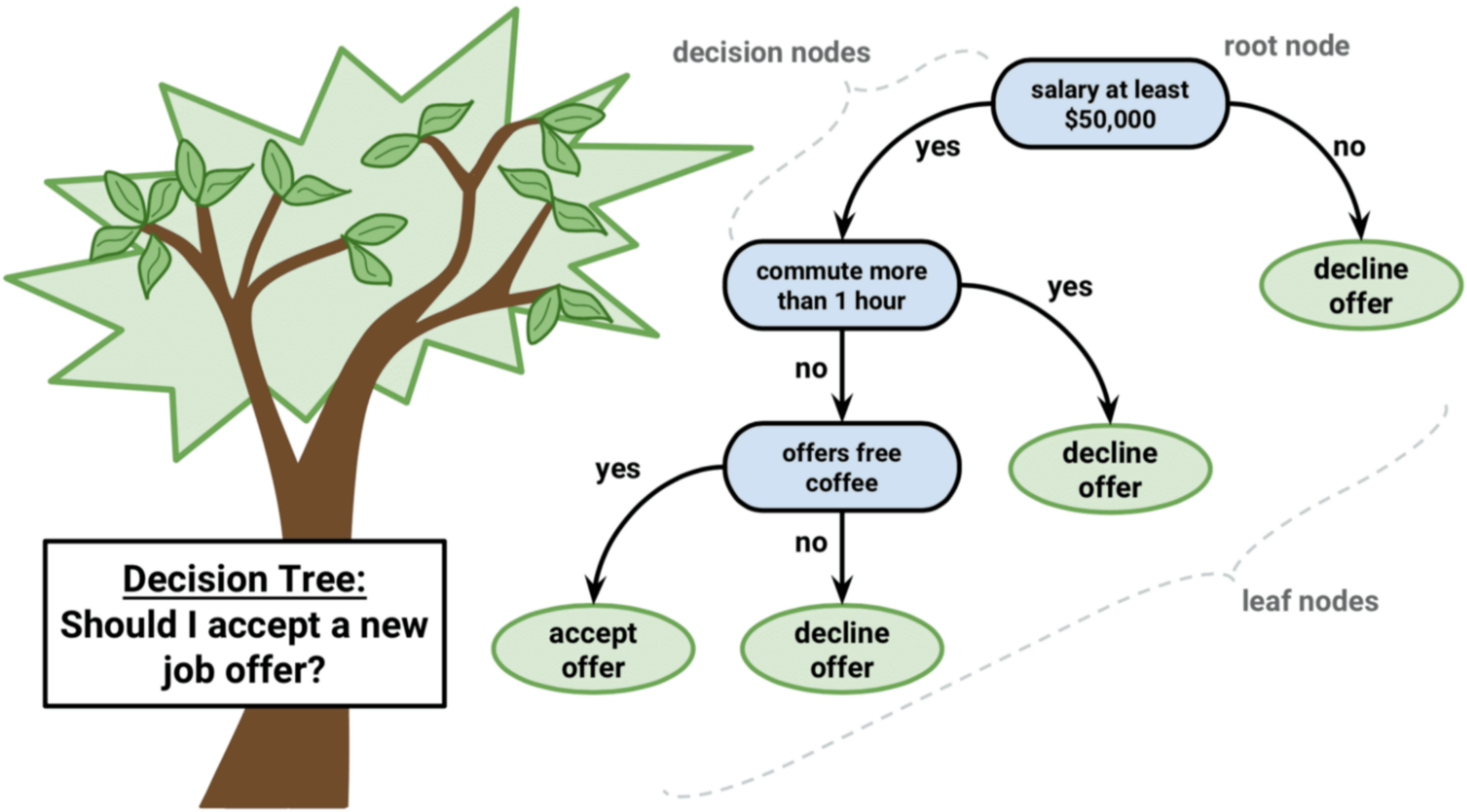
Example: Roll a die, how many dots showing on the top face? Let A = "less than 4 dots are showing."

S = { 1, 2, ... , 6 }

P = { 1/6, 1/6, ... , 1/6 }



Decision Trees for Probability Problems



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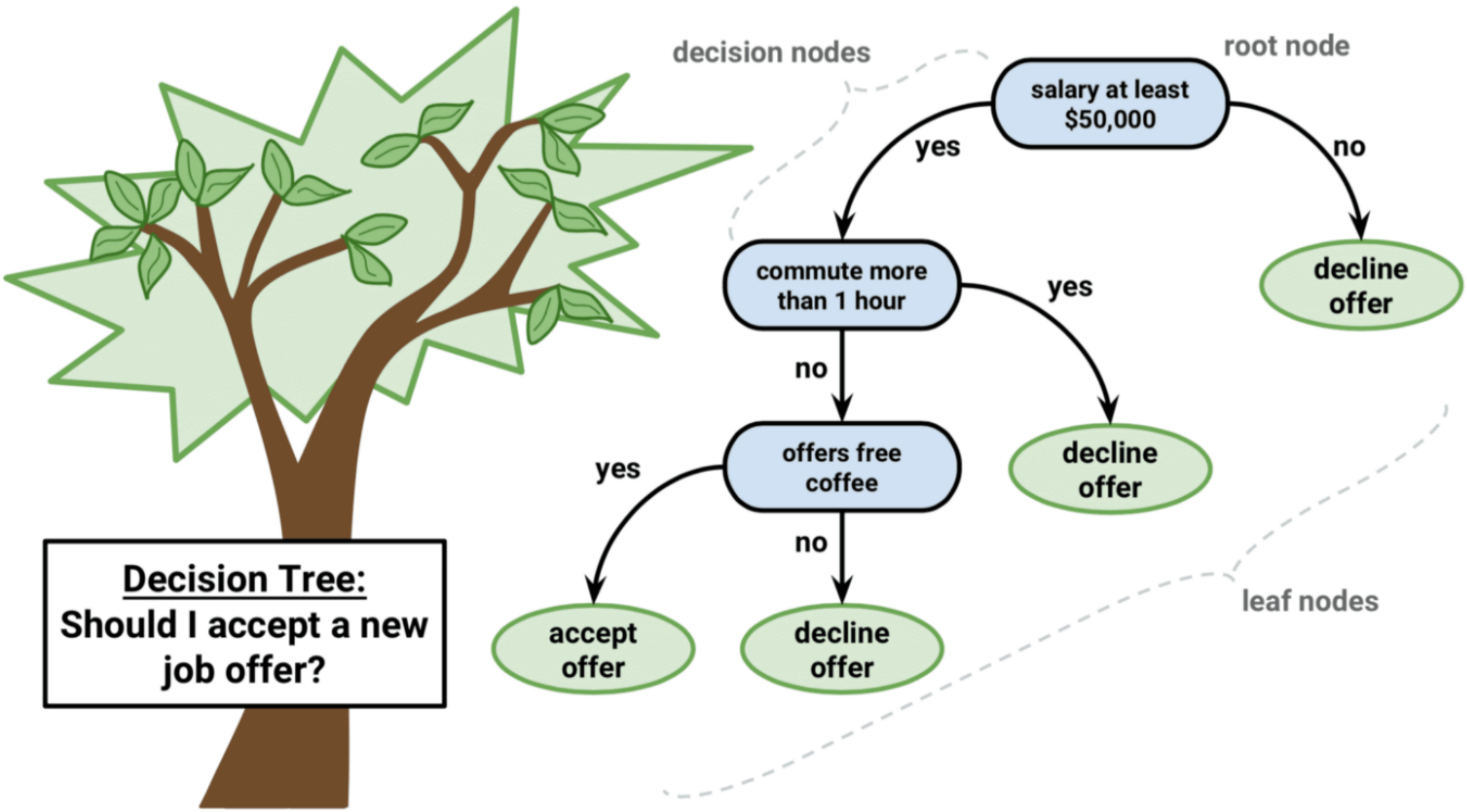

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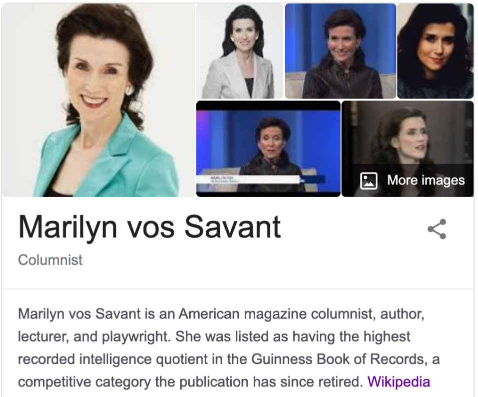
The Monte Hall Problem



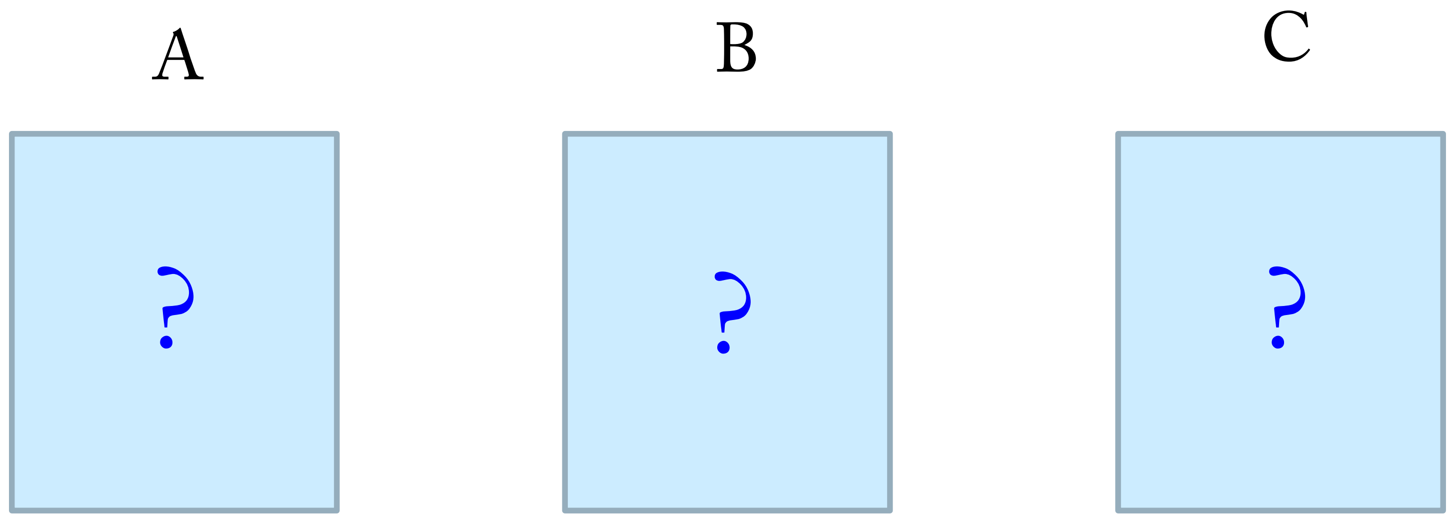
Let's Make a Deal

In the September 9, 1990 issue of *Parade* magazine, columnist Marilyn vos Savant responded to this letter:

> *Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?*

Craig. F. Whitaker
Columbia, MD

The Monte Hall Problem

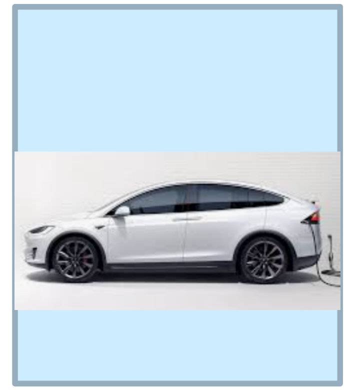


The Monte Hall Problem

A

B

C



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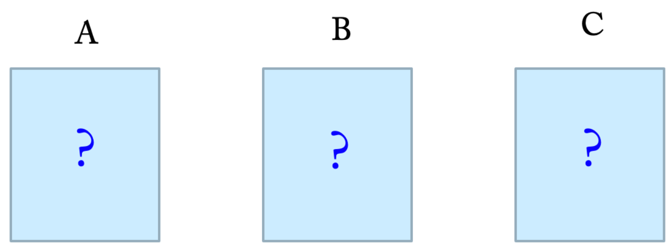

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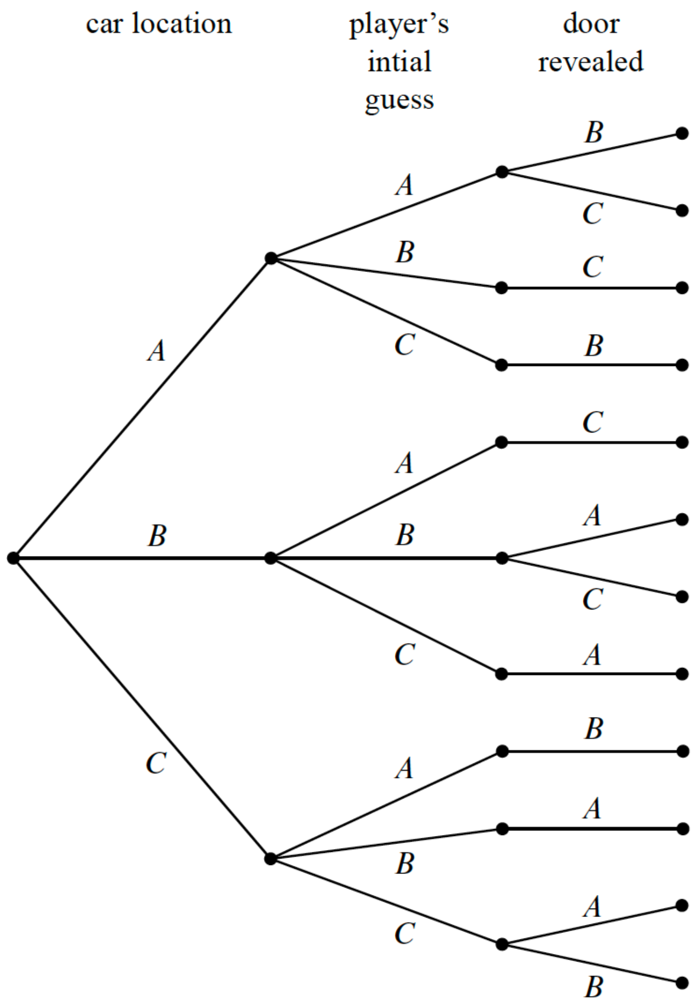
Implicit Assumptions (@DamnWordProblems)

1. The car is equally likely to be hidden behind each of the three doors.
2. The player is equally likely to pick each of the three doors, regardless of the car's location.
3. After the player picks a door, the host *must* open a different door with a goat behind it and offer the player the choice of staying with the original door or switching.
4. If the host has a choice of which door to open, then he is equally likely to select each of them.

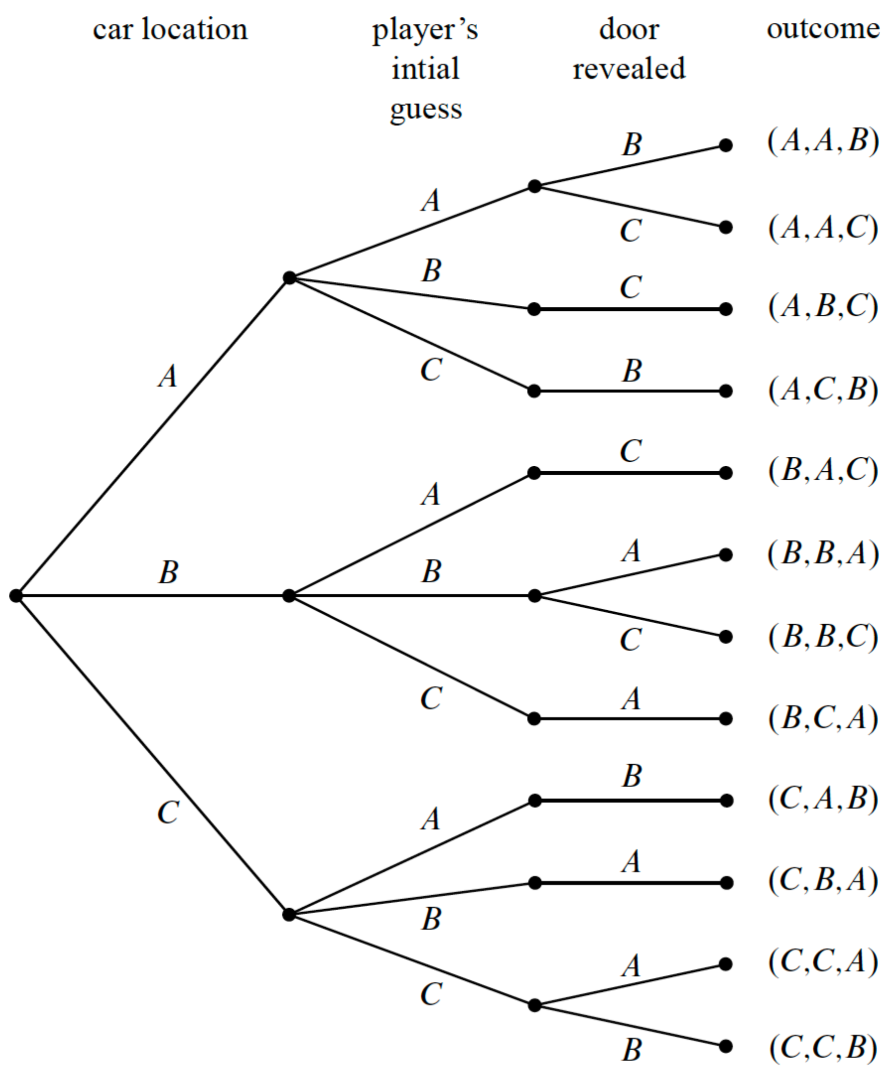
The Monte Hall Problem: The Four-Step Method

Step 1: Find the Sample Space



The Monte Hall Problem: The Four-Step Method

Step 2: Define Events of Interest



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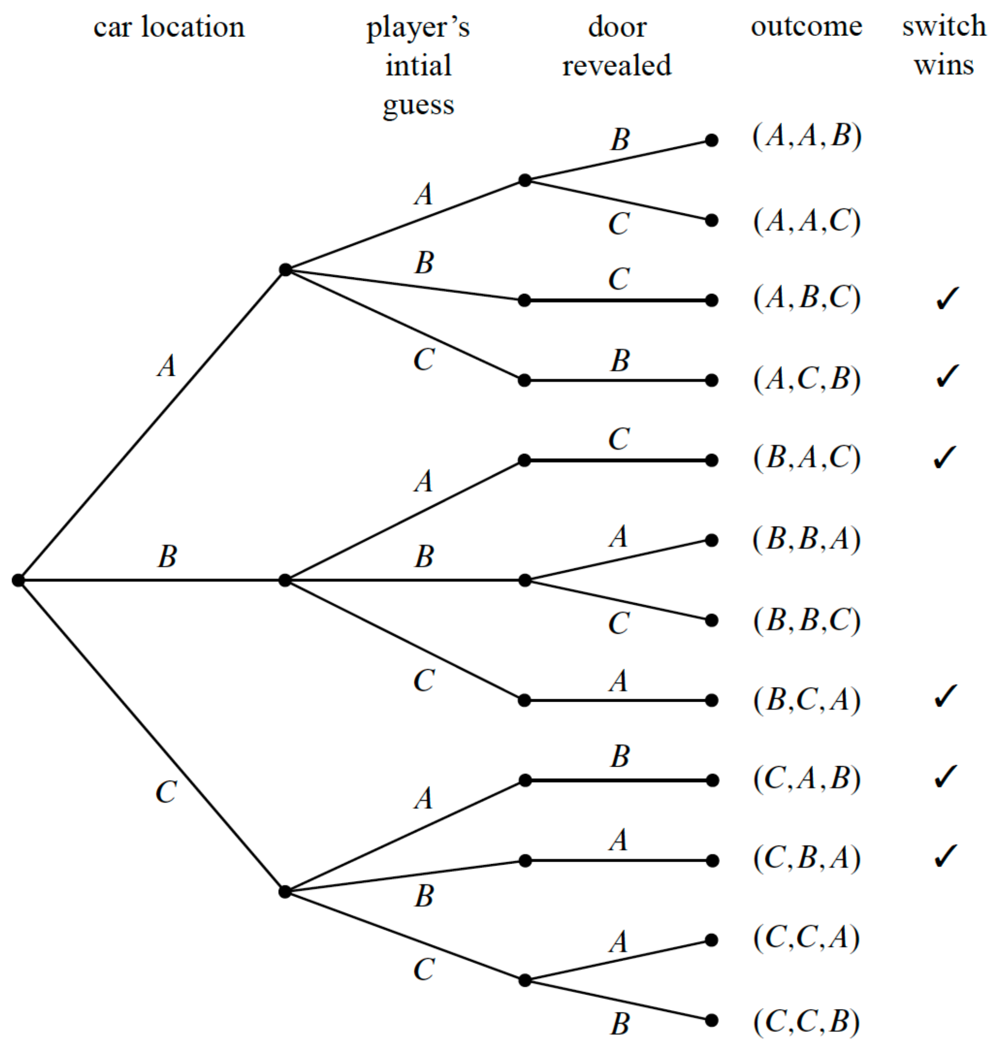
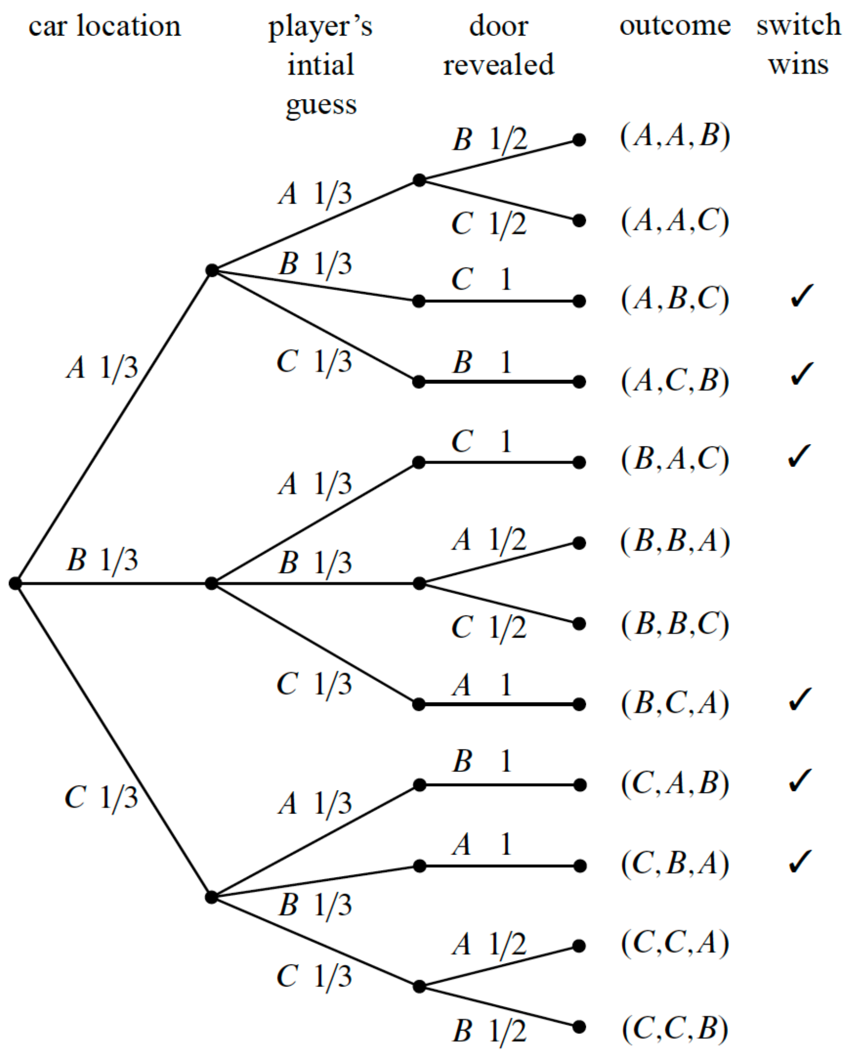


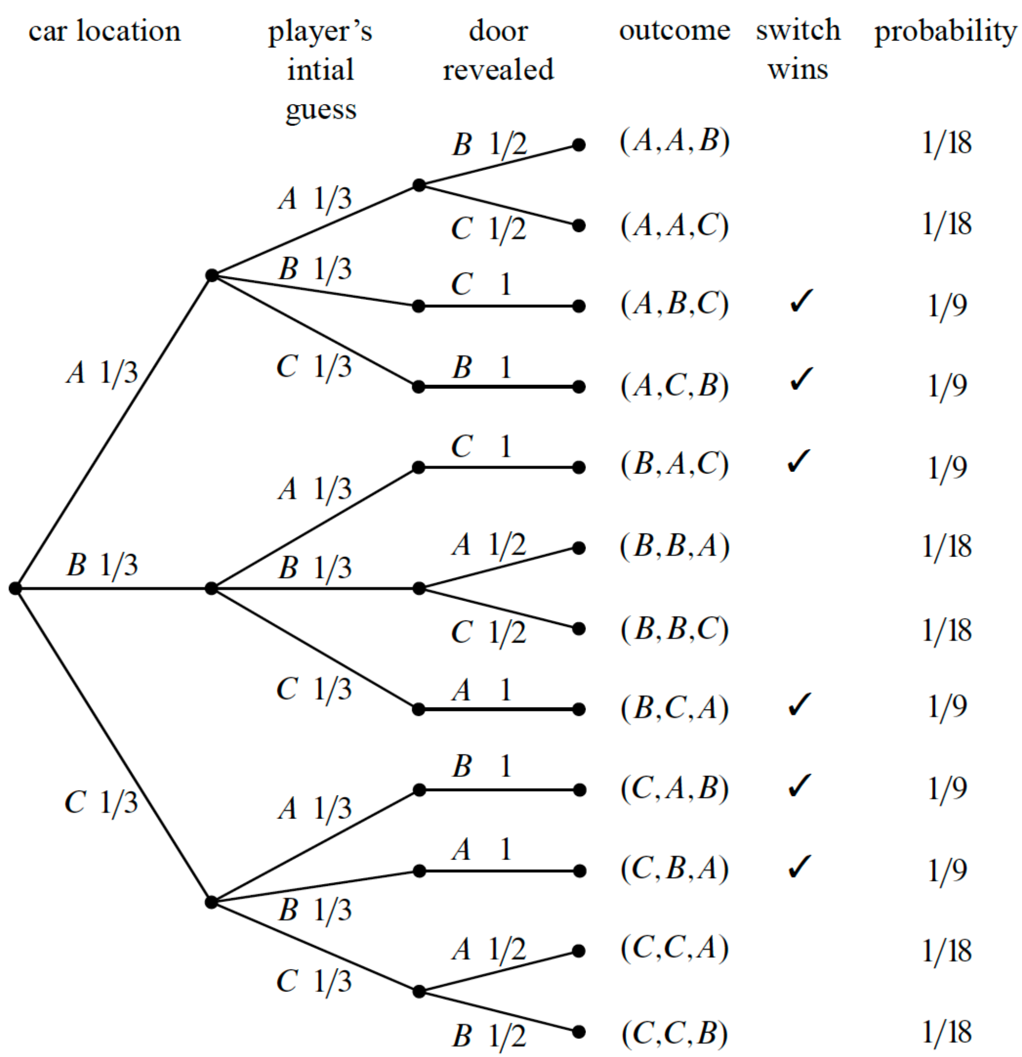
The Monte Hall Problem: The Four-Step Method

Step 3: Determine Outcome Probabilities



The Monte Hall Problem: The Four-Step Method

Step 4: Compute Event Probabilities



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